FM2 - I - 001

MATRIZEAK - MATRIZE BEREZI BATZUK

- Errenkadetako eragiketak $A \sim B$ [Eragiketen ostean ent(r)iekak desberdin dira]
- Matrize entiekak

$A \in M_{m \times n}(\mathbb{R})$ matrizearen ordena

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

errenkada → E_i (lerroa)

m → errenkadak

n → zutabeak

DETERMINANTE - KALKULUAK - TRANSFORMAZIOAK

- Metodoak $\begin{cases} n=3 \to \text{Sarrus} \\ n>3 \end{cases}$ $\sum_{i=1}^{n} a_{ij} A_{ij}$ *, $\sum_{j=1}^{n} a_{ij} A_{ij}$ *₁
- Matrizeak karratuak bakarrik dira izan
- Elementu bateko minorra, errenkada eta zutabea kenduz, geratzen diren elementuek osatzen dute.
- Adjuntua → $A_{ij} = (-1)^{i+j} M_{ij} = \pm M_{ij}$

Orokorrean negatiboko elementuen adjuntuak positiboak izango dira.

*₁ Determinantea bateko lerro edo zutabe bateko elementuak zutik beraien adjuntuekin biderkatuz eta gero eranitzak batuz emaitza lortzen dugu.

Konbinazio linealak

- $a, b, c, \ldots \in \mathbb{R}$

$aE_1 + bE_2 + \ldots$

- $E_i \pm E_j$

$2E_i + E_j$

! Bi errenkada berdinak izanez gero, determinantearen balioa nulua da.

Eragiketak

1) $E_i \leftrightarrow E_j$ errenkaden arteko trukaketa → $\det A \Rightarrow -\det A$

2) $\forall \lambda \in \mathbb{R}$; $|\lambda E_i|$ errenkadaren bat zenbaki batekin biderkatuz → $\det A \Rightarrow \lambda \cdot \det A$

] determinantea aldatu egiten da

3) $E_i \to E_i \pm [\lambda_i E_j + \lambda_n E_k + \ldots]$ beste errenkada bat gehituz/kenduz (konbinazio lineala) → $\det A = \det A$

determinantea ez da aldatzen

Rouche - Frobenius

Heineren arabera sailkapena.

- $h(A)$ → Matrize etaia

$h(A_A)$ → Matrize zabaldua

$h(A) = h(A_A)$ $\begin{cases} = n & \text{Bateragarri determinatua} \\ < n & \text{Bateragarri indeterminatua} \end{cases}$

$h(A) \neq h(A_A)$ Bateraezina

Matrize alderantzgarriak

- $A \in M_{n \times n}(\mathbb{R}) \;/\; \exists B \in M_{n \times n}(\mathbb{R})$

$A \cdot B = B \cdot A = I_n$ $\quad B = A^{-1}$

matrize singularra → $\det = 0$

! Ez dira alderantzgarriak

- A → matrize erregularra ($\det \neq 0$)

- Kalkuluak

1) $A^{-1} = \frac{1}{\det(A)} \cdot \tilde{A}$ Matrize adjuntua $\tilde{A} = (A_{ij})^{t}$

2) Errenkadetako eragiketak → $A/I_n \to I_n/A^{-1}$

3) ELS bateragarria

4) $P(A) = 0$

FM2 - J - 002

$$A = \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 \end{array}\right) \sim \begin{array}{c} \frac{1}{2}C_1 \\ \frac{1}{3}C_2 \\ \frac{1}{4}C_3 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/4 \end{array}\right)$$

$$\downarrow A^{-1}$$

$\boxed{\text{2) prozedura}}$

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}^T = \underbrace{\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix}}_{\tilde{A}} \frac{1}{2} = \underbrace{\begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & 1/2 \end{pmatrix}}_{A^{-1}}$$

$\boxed{\text{1) prozedura}}$

$A^3 - 3A^2 + 4A - 2I_3 = [0]_{3\times3}$

- I_3 askatu
- A biderkagai bezala askatu
- $A[\quad] = I_3$
- A^{-1} kalkulatzeko formula zentratu da

$\frac{A^3 - 3A^2 + 4A}{2} = I_3$ $\quad A\left[\frac{1}{2}(A^2 - 3A + 4I)\right] = I_3$

$2A^{-1} = A^2 - 3A + 4I$ $\quad A^{-1} = \frac{1}{2}[A^2 - 3A + 4I]$

$\boxed{\text{4) prozedura}}$

adib

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \qquad A^2 = \begin{pmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 3 & 0 & -3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & 1/2 \end{pmatrix} = A^{-1}$$

$\boxed{\text{Eragiketak alderantzizkoekin}}$

$A \cdot B = C \Rightarrow A \cdot B \cdot B^{-1} = C \cdot B^{-1} \Rightarrow A = C \cdot B^{-1}$

$\Rightarrow A^{-1} A \cdot B = A^{-1} C \Rightarrow B = A^{-1} C$

$A\, B\, C = D \Rightarrow A\,(BC) = D \Rightarrow A(BC)(BC)^{-1} = D\,(BC)^{-1} \Rightarrow A = D\,(BC)^{-1}$

$\Rightarrow A^{-1} A\, B\, C\, C^{-1} = A^{-1} D\, C^{-1} \Rightarrow B = A^{-1} D\, C^{-1}$

$\Rightarrow (AB)\, C = D \Rightarrow (AB)(AB)^{-1} C = (AB)^{-1} D \Rightarrow C = (AB)^{-1} D$

$(A\, B)^T = B^T A^T \qquad (A^T)^T = A$

$(A \cdot B)^{-1} = B^{-1} A^{-1} \qquad (A^{-1})^{-1} = A$

FM2 - I - 003

$$\begin{array}{ccc} A & x & y \\ \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} & \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = & \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \end{array}$$

$A \cdot x = y$

$x = A^{-1} y$

$\boxed{3) \text{ procedura}}$

$$\begin{cases} x_1 - x_3 = y_1 \\ x_2 = y_2 \\ x_1 + x_3 = y_3 \end{cases}$$

$$A/A_f = \left(\begin{array}{ccc|c} 1 & 0 & -1 & y_1 \\ 0 & 1 & 0 & y_2 \\ 1 & 0 & 1 & y_3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{y_1+y_3}{2} \\ 0 & 1 & 0 & y_2 \\ 0 & 0 & 1 & y_3 - \frac{y_1+y_3}{2} \end{array}\right)$$

$x_1 = \frac{y_1+y_3}{2}$

$x_2 = y_2$

$x_3 = y_3 - \left(\frac{y_1+y_3}{2}\right) = y_3 - x_1 = \frac{1}{2}y_3 - \frac{1}{2}y_1$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & 1/2 \end{pmatrix}}_{A^{-1}} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

Ortogonale $\Rightarrow A^T = A^{-1}$

$\det(A) = \det(A^{-1})$

$\det(A^{-1}) \det(A) = 1$

$\det(A)^2 = 1$

$\det A = \pm 1$

$\det(A) = \det(A^T)$

$\det(AB) = \det(A) \cdot \det(B)$

$\det(A) = \kappa \quad \det(A^{-1}) = \frac{1}{\kappa}$

Idempotente $\to A^2 = A$

Nilpotente $\to \det = 0$

$(A+B)^3 (A-B) = A - B \Rightarrow (A+B)^2 (A+B)(A-B) = A-B \Rightarrow (A+B)^2 (A^2 - B^2) = A - B \Rightarrow (A+B)^2 (A-B) = A-B$

$(A^3 + A^2B + AB^2 + B^3)(A-B) = A-B$

$(A + AB + B)(A-B) = A - B$

$A + \cancel{AB} + 2AB - 2AB + \cancel{BA} - B = A - B$

$\boxed{A - B = A - B}$

$(A^2 + B^2 + 2AB)(A^2 - B^2)$

$A - AB + AB - B + 2AB - 2AB = A - B$

$A - B = A - B$

$(A+B)(A+B)(A-B) = A - B$

$(A+B)(A^2 - B^2) = A - B$

$(A+B)(A-B) = A+B$

$(A^2 - B^2) = A - B$

$A - B = A - B$

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{pmatrix} = \begin{pmatrix} A_{11}A_{11} + A_{12}A_{12} & A_{11}A_{21} + A_{12}A_{22} \\ A_{21}A_{11} + A_{22}A_{12} & A_{21}A_{21} + A_{22}A_{22} \end{pmatrix}$$

$(A A^T)$ simmetrica $\quad (A A^T)^T = (A^T)^T A^T = A A^T$

A e B simmetrica $\Rightarrow A = A^T \quad A = A^T \quad B = B^T$

$(AB)^T = (AB)^T$

FM2-II004

$A^2 = [0]_{n\times n}$ $\quad$ A singular

$[\mathrm{Det}\,A]^2 = 0 \quad\quad |A| = 0$

$\Downarrow$

$\mathrm{Det}\,A = 0$

A ortogonala

$\downarrow$

$A^T = A^{-1} \Rightarrow A\cdot A^T = A^T\cdot A = I$

$A\,A^{-1} = A^{-1}A = I$

$A\cdot A^T = A^T\cdot A$

$P = A^{-1}A^T$ ortogonala dela frogatu.

$P\,P^T = I \;,\; A^{-1}A^T P^T = A^{-1}A^T(A^T)^T\cdot(A^{-1})^T = A^{-1}A^T A\cdot(A^T)^{-1} = A^{-1}A\cdot A^T\cdot(A^T)^{-1}$

$(A^{-1}A^T)^T \quad (A\cdot B)^T = B^T\cdot A^T$

$I \cdot I = I$

ERAGIKETAK ALDERANTZIKOEKIN

$A\cdot B = C \to A B B^{-1} = C\cdot B^{-1} \to A = C B^{-1}$

$\to A^{-1}AB = A^{-1}C \to B = A^{-1}C$

$ABC = D \to A(BC) = D \to A(BC)(BC)^{-1} = D(BC)^{-1} \to A = D(BC)^{-1}$

$\to A^{-1}ABCC^{-1} = A^{-1}DC^{-1} \to B = A^{-1}DC^{-1}$

$\to (AB)C = D \to (AB)(AB)^{-1}C = (AB)^{-1}D \to C = (AB)^{-1}D$

$(AB)^T = B^T A^T \quad (A^T)^T = A$

$(AB)^{-1} = B^{-1}A^{-1} \quad (A^{-1})^{-1} = A$

$\det(AB) = \det A \cdot \det B \quad\quad A A^{-1} = A^{-1}\cdot A = I$

$\det A = 3$

$\det A^{-1} = ?$

$(\det A^{-1})(\det A) = \det I$

$\det A^{-1}\cdot 3 = 1 \quad \det A^{-1} = 1/3$

$A^T = A^{-1} \to$ Matrize ortogonala

$\det A?$ $\quad A\cdot A^T = I \quad \det A^T \det A = 1$

$\det A = \det A^T \;\Big|\; (\det A)^2 = 1 \quad \det A = \pm 1$

FM2 . J - 003

<u>Ariketak</u>

$A, B \in M_{n\times n}(\mathbb{R})$ / $A\cdot B = B\cdot A$

Idenpotenteak $\left\{ \begin{array}{l} A^2 = A \\ B^2 = B \end{array} \right.$

Frogatu: $(A+B)^3(A-B) = A-B$

$$(A+B)^2(A+B)(A-B) = A-B$$

$$(A^2+2AB+B^2)(A^2-B^2) = A-B$$

$$(A+2AB+B)(A-B) = A-B$$

$$\cancel{A^2} - \cancel{AB} + 2\cancel{A^2}B - 2\cancel{AB^2} + \cancel{AB} - B^2 = A-B$$

$$\boxed{A-B = A-B}$$

$A, B \in M_{n\times n}(\mathbb{R})$ Frogatu:

- $A\,A^T$ simetrikoa dela

$$A\,A^T = (A\,A^T)^T = A^T\cdot(A^T)^T = A^T\cdot A$$

- A eta B simetrikoak $\Rightarrow (A+B)^n$ simetrikoa $\forall n \in \mathbb{N}$

$A = B^T \quad B = A^T$

$(A+B)^n = (B^T+A^T)^n = \left[(B+A)^T\right]^n = \left[(A+B)^n\right]^T$

- A eta B antisimetrikoak $\to$ $\forall n$ bikoiti $\in \mathbb{N} \to (A+B)^n$ simetrikoa
$\forall n$ bakoiti $\in \mathbb{N} \to (A+B)^n$ antisimetrikoa

$A^T = B \quad B^T = A$

$A = -A^T \to -A = A^T$

$B = -B^T \to -B = B^T$

$$\left[(A+B)^n\right]^T = \left\{\left[-1(A^T+B^T)\right]^n\right\} = (-1)^n\left[(A+B)^{T\cdot n}\right]^T$$

$$= (-1)^n(A+B)^n$$

$$\left[(A+B)^n\right]^T = \left\langle \begin{array}{l} n = 2k \to (A+B)^n \\ n = 2k-1 \to -(A+B)^n \end{array} \right.$$

- A eta B antisim.: $A\cdot B$ sim $\Leftrightarrow A\cdot B = B\cdot A$

$A^T = -B \quad B^T = -A \quad (A\,B)^T = \left[(-B^T)\cdot(-A^T)\right]^T = (B^T A^T)^T = \left[(B\cdot A)^T\right]^T = B\cdot A$

$A\cdot B$ sim $\to (A\cdot B)^T = A\,B$

$\Big\}\; A\cdot B = B\cdot A$

FM2-J-006

$A \in M_{n\times n}(\mathbb{R})$ edozein matrize nilpotente singularra dela frogatu.

$A^p = [0]_{n\times n}$ $\quad \det(A^p) = \det[0]_{n\times n} = 0$

$(\det A)^p = \det[0]_{n\times n} \quad (\det A)^p = 0 \to \det A = 0$

V $+$ . . → Kanpo konposaketa legea

↓

Barne konposaketa legea

$\forall \vec{u} \wedge \vec{v} \in V$

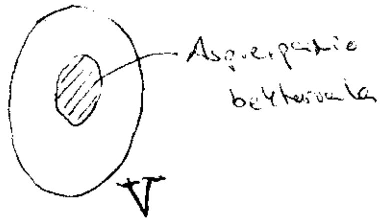
$\vec{u} + \vec{v} \in V$

$\forall \lambda \in \mathbb{R} \longrightarrow \lambda \vec{u} \in V$

$\forall \lambda \notin \mathbb{K}$

$\lambda \cdot \vec{u} + \mu \cdot \vec{v} \in V$ → Konbinazio lineala

V
- $\mathbb{R}^n$, $n \in \mathbb{N}$ $\begin{cases} \mathbb{R}^2 \Rightarrow (a,b) & a,b \in \mathbb{R} \\ \mathbb{R}^3 \Rightarrow (a,b,c) & a,b,c \in \mathbb{R} \\ \mathbb{R}^4 \Rightarrow (a,b,c,d) & a,b,c,d \in \mathbb{R} \end{cases}$
- $P_n;\ \forall \vec{v} \in P_n \,/\, \vec{v} = P_n(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$
- $M_{n\times n}(\mathbb{R})$
- C, $[a,b]$ jarraia



F: Bektore-sistema → Sistema lotua / Sistema askea

Bektore-sistema baten heina: $h(F)$

Adib.

$F = \{(1,2)\ (2,3)\} \subset \mathbb{R}^2$ → bektore espazioa

$h(F) = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = 2$ → Zenbat bektore linealki independente dituen $\Rightarrow h(F) = 2 = m_K \to$ sistema askea

$G = \{(1,2), (2,4)\} \subset \mathbb{R}^2$

$h(G) = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} = 1 \Rightarrow h(G) = 1 < m_K \to$ sistema lotua $\qquad m_K$: bektore kopurua

$M = \{x^2, 3-2x, x^2+2x-3\} \subset P_2 \Rightarrow M = \{(0,0,1), (3,-2,0), (-3,2,1)\}$

$h(M) = \begin{vmatrix} 0 & 0 & 1 \\ 3 & -2 & 0 \\ -3 & 2 & 1 \end{vmatrix} = 2 \Rightarrow h(M) = 2 < m_K \to$ sistema lotua

FM2 - 5 - cot

$$H = \left\{ \begin{pmatrix} 2 & 3 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} -3 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 7 & 6 \\ -1 & 0 \end{pmatrix} \right\}$$

$$H' = \{(2,3,0,0), (-3,0,1,0), (7,6,-1,0)\}$$

$$h(H) = \begin{pmatrix} 2 & 3 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ 7 & 6 & 1 & 0 \end{pmatrix} = 2 < m_{ex}$$ sistema lotua

1.2 Ariketa

a) Azpiespazio bektorialak dira?

$S = \{(0,a,b) \,/\, a,b \in \mathbb{R}\} \subset \mathbb{R}^3$

$(0,a,b) = (0,a,0) + (0,0,b) = a(0,1,0) + b(0,0,1)$

$\vec{v} \in S$: $\vec{u} = (0,a_1,b_1)$
$\vec{v} = (0,a_2,b_2)$

$\vec{u}+\vec{v} = (0, a_1+a_2, b_1+b_2) \subset S$? Bai, beraz azpiespazio bektoriala.

$T = \{(0,a,1) \,/\, a \in \mathbb{R}\}$

$\vec{u} = (0,0,1) \in T$
$\vec{v} = (0,1,2) \notin T$
$\vec{w} = (0,2,1) \in T$

$\vec{u}+\vec{w} = (0,2,2) \notin T$

$\vec{u} = (0,a_1,1)$; $\lambda\vec{u} = \lambda(0,a,1)$
$\vec{v} = (0,a_2,1)$

$\vec{u}+\vec{v} = (0,a_1+a_2,2) \notin T$

Ez da azpiespazio bektoriala

$Z = \{(x,y,z) \,/\, x+y=0 \wedge x-y=z\}$

$\vec{u} = (x_1,y_1,z_1)$
$\vec{v} = (x_2,y_2,z_2)$

$\vec{u}+\vec{v} = (\underbrace{x_1+x_2}_{x_b}, \underbrace{y_1+y_2}_{y_b}, \underbrace{z_1+z_2}_{z_b}) \in Z \begin{cases} x_b+y_b=0 \\ x_b-y_b=z_b \end{cases}$

$\vec{u} \in Z$: $x_1+y_1=0 \wedge x_1-y_1=z_1$
$\vec{v} \in Z$: $x_2+y_2=0 \wedge x_2-y_2=z_2$

$(x_1+x_2)(y_1+y_2)=0$ $\quad (x_1+x_2)-(y_1+y_2)=z_1+z_2$

Azpiespazio bektoriala da: $x_b+y_b=0$ $\quad x_b-y_b=z_b$

$\lambda(x,y,z) \in Z$?

$(\lambda x, \lambda y, \lambda z) \in Z$ $\quad \lambda x+\lambda y=0 \quad \lambda(x+y)=0 \quad x+y=0$
$\lambda x-\lambda y=\lambda z \quad \lambda(x-y)=\lambda z \quad x-y=0$

Bektore-sistema baten itxidura lineala

$F = \{(1,0,0),(0,1,0)\}$

$h(F) = h\begin{pmatrix}1&0&0\\0&1&0\end{pmatrix} = 2 \rightarrow$ Sistema askea

$\{(1,1,0),(3,2,0),(2,3,0)\ldots\}$ — Bektoreekin, konbinazio linealak bateter, itxidura linealen sortu daitekegu. Itxidura lineala multiz baten da.

S-ren oinarria

Azpiespazio bektoriala — $S = \mathcal{L}(F) = \mathcal{L}(B_s)$

S-ren sortzaileak
Sistema-sortzaileak

S askea → S-ren oinarria
S, lotua

Bektore kopuruaz kardinal' dei

$h(B_s) = h = \dim S$

Heina: $h(F)$ $\begin{cases} h(F) = m & \text{askea} \\ h(F) < m & \text{lotua}\end{cases}$

Bektore linealki independenteen kopurua — Bektore kopurua

$\dim S < \dim V$
↳ V-ren azpiespazio propioa da S

$\dim S = \dim V$
↳ $S = V$

$\dim S \geq 1$
↳ Dimentsioa beti izango da handiago edo berdin bat

$S = \{(x_1, x_2, x_3) / x_1 = 2x_2 + x_3\}$ $\qquad h(F) = h\begin{pmatrix}2&1&0\\1&0&1\end{pmatrix} = 2 =$ bekt kop

$\vec{u} = (2x_2 + x_3, x_2, x_3)$ $\quad \vec{u} = (2,1,0) \in S$, $\vec{v} = (1,0,1) \in S$ $\begin{cases} \vec{u}+\vec{v} \in S? & (3,1,1) \text{ Bai ✓} \\ \lambda\vec{u} \in S? & (4,2,0) \text{ Bai ✓}\end{cases}$ $\quad S = \mathcal{L}(F)$

$(2x_2, x_2, 0) + (x_3, 0, x_3) \Rightarrow x_2(2,1,0) + x_3(1,0,1) \Rightarrow F = \{(2,1,0),(1,0,1)\} \rightarrow$ Sortzailea

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$T = \{(x_1, x_2, x_3) / 2x_1 + x_2 = 3x_1 + x_3 = 0\}$ $\quad x_1 = \frac{-x_2}{2}$ $\quad x_1 = \frac{-x_3}{3}$ $\quad x_2 = -2x_1$ $\quad x_2 = \frac{2x_3}{3}$ $\qquad \mathcal{L}(F) = T \qquad \dim T =$

$\vec{u} = x_3(-1/3, 2/3, 1)$ $\quad$ $n = 2$ ekuazio → 2 ezezagun ezagun
3 ezezagun → 1 ezezagun askea $\qquad F = \{(-1/3, 2/3, 1)\}$

$W = \{(1, m, n) / m, n \in \mathbb{R}\}$ ✗ $\quad (1,0,0) + m(0,1,0) + n(0,0,1) \quad F = \{(1,0,0),(0,1,0),(0,0,1)\}$

$S = \{(x_1, x_2, x_3, x_4) / x_3 = 0 \wedge x_1 + x_2 + x_4 = 0\}$ $\quad \vec{u} = (-x_2 - x_4, x_2, 0, x_4)$
$x_2(-1,1,0,0) + x_4(-1,0,0,1)$

$h(F) = h\begin{pmatrix}-1&1&0&0\\-1&0&0&1\end{pmatrix} = 2 =$ bekt kop $\qquad F_s = \{(-1,1,0,0),(-1,0,0,1)\}$

$S = \mathcal{L}(F)$

$\dim S = 2$

M2-3-00G

$F=\left\{\begin{pmatrix}1&1\\0&0\end{pmatrix},\begin{pmatrix}0&1\\1&0\end{pmatrix},\begin{pmatrix}3&0\\1&0\end{pmatrix}\right\}$ Sistema aske edo lotua da?

$S=\underbrace{\mathcal{L}(F)}_{\text{Sortzailea}}$ ren oinarri bat eta dimentsioa

$\{(1,1),(0,0)\}$
$\{(0,1),(1,0)\}$
$\{(3,0),(1,0)\}$

$$h(F)=h\begin{pmatrix}1&1&0&0\\0&1&1&0\\3&0&1&0\end{pmatrix}=3$$ = bektore kopurua
Sistema askea

dim = 3

Sortzaile eta aske = oinarria

$G=\left\{\begin{pmatrix}1&2\\0&0\end{pmatrix},\begin{pmatrix}0&1\\1&0\end{pmatrix},\begin{pmatrix}3&5\\-1&0\end{pmatrix}\right\}$

$$h(G)=h\begin{pmatrix}1&2&0&0\\0&1&1&0\\3&5&-1&0\end{pmatrix}\sim\begin{pmatrix}1&2&0&0\\0&1&1&0\\3&6&0&0\end{pmatrix}=2$$ ≠ bektore kopurua
Sistema lotua

det = 0

$T=\mathcal{L}(G)$

$\dim T=2 \qquad B_T=\left\{\begin{pmatrix}1&2\\0&0\end{pmatrix},\begin{pmatrix}0&1\\1&0\end{pmatrix}\right\}=$

$S\in\mathbb{R}^2 \qquad B_S\to B_{\mathbb{R}^3}$?
$Z\in\mathbb{R}^3 \qquad B_Z\to B_{\mathbb{R}^3}$?

B_S oinarri baten hedapena → B_V
↳ S azpiespazio baten oinarria
S ⊂ V → Bektore-espazioa

V-ren oinarri bat
↳ Sistema aske

$B_S=\{(0,1,0),(0,0,1)\}\to B'_{\mathbb{R}^3}=\{(0,1,0),(0,0,1),(1,0,0)\}$

$$h(B^1_{\mathbb{R}^3})=h\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=3$$

$B_Z=\{(1,-1,2)\}\to B^2_{\mathbb{R}^3}=\{(1,-1,2),(0,1,0),(0,0,1)\}$

$$h(B^2_{\mathbb{R}^3})=h\begin{pmatrix}1&-1&2\\0&1&0\\0&0&1\end{pmatrix}=3$$

Hedapena → bektoreak gehitu matrike koadratuko eta lerroi dagon heinean(?)

FM2-5 = 0.10

1.3 Antiteka

$S = \{(x_1, x_2, x_3, x_4) \mid x_3 = 0 \wedge x_1 + x_2 + x_3 + x_4 = 0\}$

$B_S = \{(1,-1,0,0), (0,-1,0,1)\} \Rightarrow B_{\mathbb{R}^4} = \{(1,-1,0,0), (0,-1,0,1), (0,0,1,0), (0,0,0,1)\}$

$$r(B_{\mathbb{R}^4}) = r\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = 4$$

$S \subset \mathbb{R}^4 \to \dim \mathbb{R}^4 = 4$ lastnosti vekt. kap.

$B_S = \{(1,-1,2,2), (1,2,3,4)\} \to B^S_{\mathbb{R}^4} = \{(1,-1,2,2), (1,2,3,4), (0,0,1,0), (0,0,0,1)\}$

$$r(B^S_{\mathbb{R}^4}) = 4 = r\begin{pmatrix} 1 & -1 & 2 & 2 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \overset{E_2 - E_1}{=} r\begin{pmatrix} 1 & -1 & 2 & 2 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$S_r = \{(1,-1,0,2), (1,2,0,4), (3,2,0,-3)\} \to S^r_{\mathbb{R}^4} = \{(1,-1,0,2)\ (1,2,0,4)\ (3,2,0,-3), (\}$

$$\cancel{r(S^r_{\mathbb{R}^4}) = r\begin{pmatrix} 1 & -1 & 0 & 2 \\ 1 & 2 & 0 & 4 \\ 3 & 2 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix} = r\begin{pmatrix} 1 & -1 & 0 & 2 \\ 0 & 3 & 0 & 2 \\ 0 & 5 & 0 & -9 \\ 0 & 0 & 1 & 1 \end{pmatrix}}$$

Dimenzija od ?

Baza, $r(S^r_{\mathbb{R}^4})$ = vekt. kapica

$$r(S^r_{\mathbb{R}^4}) = r\begin{pmatrix} 1 & -1 & 0 & 2 \\ 1 & 2 & 0 & 4 \\ 3 & 2 & 0 & -3 \end{pmatrix} = r\begin{pmatrix} 1 & -1 & 2 \\ 1 & 2 & 4 \\ 3 & 2 & -3 \end{pmatrix} = r\begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 2 \\ 0 & 5 & -9 \end{pmatrix} = 3$$

$B^r_{\mathbb{R}^4} = \{(1,-1,0,2), (1,2,0,4), (3,2,0,-3), (0,0,1,0)\}$

$$\begin{pmatrix} 1 & -1 & 0 & 2 \\ 1 & 2 & 0 & 4 \\ 3 & 2 & 0 & -3 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

M2-5-011

1.1 Ariketa

- $B=\{x^2+x+1,\ x^2+x,\ x^2\}$
  $\downarrow \quad \downarrow \quad \downarrow$
  $\bar{e}_1 \quad \bar{e}_2 \quad \bar{e}_3$

$$h(B_{P_2})=h\begin{array}{ccc} x & 1 & x^2 \\ \downarrow & \downarrow & \downarrow \end{array}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}=3$$

$B=B_{P_2}$?

$B=\{(x,1,0),(0,0,x+2),(1,1,1)\}$ $\quad x\in\mathbb{R}? \iff B'=B_{\mathbb{R}^3}$

$$h(B')=h\begin{pmatrix} x & 1 & 0 \\ 0 & 0 & x+2 \\ 1 & 1 & 1 \end{pmatrix}^{A_{B'}} \qquad \det(A_{B'})=\begin{vmatrix} x & 1 & 0 \\ 0 & 0 & x+2 \\ 1 & 1 & 1 \end{vmatrix}=-(x+2)\begin{vmatrix} x & 1 \\ 1 & 1 \end{vmatrix}=-(x+2)(x-1)=0$$

$x+2=0 \quad x=-2$
$x-1=0 \quad x=1$

1. kasu $\quad x\in\{-2,1\} \rightarrow h(A_{B'})=2 \quad$ Ez

2 kasu $\quad x\in\mathbb{R}-\{-2,1\} \rightarrow h(A_{B'})=3 \quad$ Bai, oinarria da

---

$S=\{(x_1,x_2,x_3,x_4)\ /\ x_1=x_2\}$, $\quad T=\mathcal{L}(\{(1,1,2,1),(2,0,-1,1)\})=\mathcal{L}(S_T)$

$\mathbb{R}^3 \longrightarrow \mathcal{L}(B_c)=\mathbb{R}^3$

$\mathcal{L}(B_c)=\mathcal{L}(\{(1,0,0),(0,1,0),(0,0,1)\})$

$\bar{v}=(x_1,x_2,x_3,x_4)=(x_1,x_1,x_3,x_4)=x_1(1,1,0,0)+x_3(0,0,1,0)+x_4(0,0,0,1)$

$$h(S_S)=h\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}=3=\text{bekt. kop} \rightarrow \text{sortzaile eta oinarri}$$

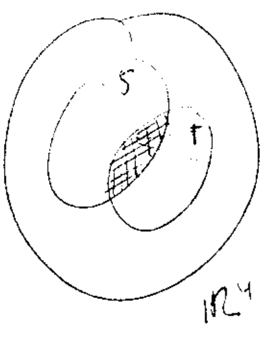
$$h(S_T)=h\begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 0 & 1 & 1 \end{pmatrix}=2=\text{bekt. kop} \rightarrow \text{sortzaile eta oinarri}$$

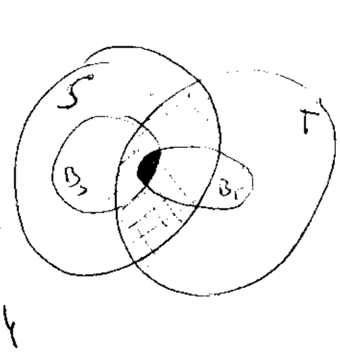
$B_S=\{(1,1,0,0),(0,0,1,0),(0,0,0,1)\}$

$B_T=\{(1,1,2,1),(2,0,-1,1)\}$
$\qquad\quad \bar{v}_T{}^{*} \qquad \bar{v}_T{}^{*}$

$$h\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 1 \end{pmatrix}=h\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 0 \end{pmatrix}=3$$

Konbinazio linealean dauden koordenatuak ez diren 0 izatea baztertu.





$B_{S\cap T}=\{\bar{v}_T\}$

$$h\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 1 & 1 \end{pmatrix}=h\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{pmatrix}=4$$

FM2-S-042

$S \subset P_3 \qquad S = \{p(x) \in P_3 \ / \ p(1) = p(-1) \wedge p'(1) = 0\}$

$p(x) = ax^3 + bx^2 + cx + d$

$$\left.\begin{array}{l} p(1) = a+b+c+d \\ p(-1) = -a+b-c+d \end{array}\right\} a+\cancel{b}+c+\cancel{d} = -a+\cancel{b}-c+\cancel{d} \Rightarrow a+c = -(a+c) \Rightarrow 2(a+c) = \ldots$$

$p'(x) = 3ax^2 + 2bx + c$

$p'(1) = 3a + 2b + c = 0 \longrightarrow 3a+2b+c = 2a+2c$

$a + 2b - c = 0$

$2a = -2c$

$a = -c \quad c = -a$

$3a + 2b - a = 0$

$2a = -2b$

$a = -b$

$-a = b$

$-3c + 2b - c = 0$

$-4c = 2b$

$-2c = b$

$p(x) = -cx^3 - 2cx^2 + cx + d$

$p(x) = ax^3 - ax^2 - ax + d$

$a(x^3 - x^2 - x) + d$

$\Updownarrow$

$(d, -a, -a, a)$

$\Updownarrow$

$d(1,0,0,0) + a(0,-1,-1,1) \Leftrightarrow (d,0,0,0) + (a,-a,-a,a)$

$h(B_s) = h\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \end{pmatrix} = 2 \qquad B_s = \{1, -x - x^2 + x^3\}$

$(V, <,>) \longrightarrow \mathbb{R}^n, P_n, M_{m\times n}(\mathbb{R}), C, [a,b)$

bektorea

biderketa eskalarra $\rightarrow$ { norma euklidearra (bektore batena) $*_1$; distantzia (bi bektoreen artean) $*_2$; proiekzioa ($\bar{x}$, $\bar{y}$ gainean) $*_3$ } $\longrightarrow$ { ortogonalketa (oinarri batean) $\downarrow$ hurbilketa lineal hobea }

$<x, x^2> = \int_{-1}^{1} x^3 dx = \left[\frac{x^4}{4}\right]_{-1}^{1} = \frac{1}{4}\left[1^4 - (-1)^4\right] = 0$

$*_1 \ \|\bar{x}\| = +\sqrt{<\bar{x},\bar{x}>}$

$*_2 \ \|\bar{x} - \bar{y}\| = d(\bar{x},\bar{y}) = +\sqrt{<(\bar{x}-\bar{y}),(\bar{x}-\bar{y})>}$

$*_3 \ P_{\bar{y}}\bar{x} = \frac{<\bar{x},\bar{y}>}{\|\bar{y}\|^2} \cdot \bar{y}$

$\cos\theta = \frac{<\bar{x},\bar{y}>}{\|\bar{x}\| \cdot \|\bar{y}\|} \qquad \cos\theta = \frac{16}{\sqrt{14}\cdot\sqrt{21}} = \frac{16}{\sqrt{6}}$

Adib

$\bar{x} = (1,2,3)$

$\bar{y} = (2,1,4)$

$\|\bar{x}\| = \sqrt{1^2+2^2+3^2} = \sqrt{14} \qquad \bar{x}-\bar{y} = (-1, 1, -1)$

$\|\bar{y}\| = \sqrt{2^2+1^2+4^2} = \sqrt{21}$

$d(\bar{x},\bar{y}) \Rightarrow \sqrt{(-1)^2+1^2+(-1)^2} = \sqrt{3}$

$P_{\bar{y}}\bar{x} \Rightarrow \frac{16}{21} \cdot (2,1,4) \Rightarrow \left(\frac{32}{21}, \frac{16}{21}, \frac{64}{21}\right)$

$P_{\bar{x}}\bar{y} \Rightarrow \frac{16}{14} \cdot (1,2,3) \Rightarrow \frac{8}{7}(1,2,3) \Rightarrow \left(\frac{8}{7}, \frac{16}{7}, 2\ldots\right)$

###

FM2-I-013

OINARRI baten ORTOGONALKETA

$B_S = (\bar u_1, \bar u_2, \dots, \bar u_r)$ / $\mathcal{L}(B_S) = S$ → azpiespazio bektoriala

$B_{og} = (\bar v_1, \bar v_2, \dots, \bar v_r)$

→ $\mathcal{L}(B^S_{og}) = S$

↓

$B_{og} = \{\bar v_1, \bar v_2, \dots, \bar v_r\}$

↓

Bektoreak binaka hartuta, elkarren arteko biderketa eskalarrak zero izan behar du.

B_C: oinarri ortogonalak

$\mathbb{R}^3$: $B_C = \{(1,0,0), (0,1,0), (0,0,1)\}$

$$\frac{(1,0,0)\cdot(0,1,0)}{0} \qquad \frac{(1,0,0)\cdot(0,0,1)}{0} \qquad \frac{(0,1,0)\cdot(0,0,1)}{0}$$

GRAM-SCHMIDT prozedura

$\bar u_1 = \bar v_1$

$$\bar v_2 = \bar u_2 - \frac{\langle \bar u_2, \bar v_1 \rangle}{\|\bar v_1\|^2}\cdot \bar v_1 = \bar u_2 - P_{\bar v_1}\bar u_2$$

$$\bar v_3 = \bar u_3 - \frac{\langle \bar u_3, \bar v_1 \rangle}{\|\bar v_1\|^2}\cdot \bar v_1 - \frac{\langle \bar u_3, \bar v_2 \rangle}{\|\bar v_2\|^2}\cdot \bar v_2 = \bar u_3 - P_{\bar v_1}\bar u_3 - P_{\bar v_2}\bar u_3$$

$$\bar v_4 = \bar u_4 - P_{\bar v_1}\bar u_4 - P_{\bar v_2}\bar u_4 - P_{\bar v_3}\bar u_4$$

$B_S = \{(1,2,0), (3,0,4)\}$ / $S = \mathcal{L}(B_S)$

Ortogonala da? $1\cdot 3 + 2\cdot 0 + 0\cdot 4 = 3$ Ez, ez da ortogonala
Ez dira ortogonalak

$B_{og} = (\bar v_1, \bar v_2) = (\bar u_1, \bar v_2)$ $\quad \bar u_1 = \bar v_1$ $\quad B_{og} = \{(1,2,0), (\frac{12}{5}, -\frac{6}{5}, 4)\}$

$$\bar v_2 = (3,0,4) - \frac{3}{5}\cdot(1,2,0) = (3,0,4) - (\tfrac{3}{5}, \tfrac{6}{5}, 0) = (\tfrac{12}{5}, -\tfrac{6}{5}, 4)$$

$S \cap T$
$S \cup T$

$S \Rightarrow B_S = \{\overset{\bar u_1}{(1,1,0,0)}, \overset{\bar u_2}{(0,0,1,0)}, \overset{\bar u_3}{(0,0,0,1)}\}$ $\qquad S \cap T = \bar 0$ edo $S \cap T \neq \bar 0$

$T \Rightarrow B_T = \{\underset{\bar v_1}{(1,1,2,1)}, \underset{\bar v_2}{(2,0,1,-1)}\}$

$B_{S\cap T} = \{w_1, w_2, \dots\} = \{\bar v_1\} = \{(1,1,2,1)\}$

$u_1 \in T$? $\quad h(\bar v_1, \bar v_2, \bar u_1) = h\begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \end{pmatrix} = 3 \Rightarrow$ ez dago konbinazio linealean

$u_2 \in T$? $\quad h(\bar v_1, \bar v_2, \bar u_2) = h\begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = 3$

$v_1 \in S$? $\quad h(\bar u_1, \bar u_2, \bar u_3, \bar v_1) = h\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 1 \end{pmatrix} = 3 \Rightarrow$ konbinazio linealean dago

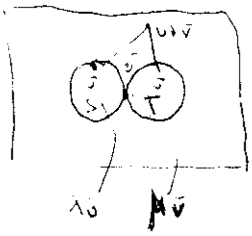
FH2 · 5 · 014

Dim - ezeroren bektore kopurua

S / $B_S = \{(1,2,0)\}$ $\bar{u}$ dim $S = 1$

T / $B_T = \{(1,0,1)\}$ $\bar{v}$ dim $T = 1$

$S \cap T = \bar{0} = (0,0,0)$



$\bar{u}+\bar{v} \in S \cup T$? $\bar{u}+\bar{v} = (2,2,1)$ Ez, kanpoan dago, ez da ez $\lambda\bar{u}$ ez $\mu\bar{v}$ notatzen.

Nola kalkulatu ezezagun batekiko bektore baten koordenatuak

$\Downarrow$

ELS $\longrightarrow$ Forma matriziala

ELS Forma bektoriala

$$\left\{\begin{array}{l} x+2y+3z=4 \\ x-y+z=2 \\ x-z=1 \end{array}\right. \qquad A^* = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & -1 & 1 & 2 \\ 1 & 0 & -1 & 1 \end{pmatrix}$$

$x(1,1,1) + y(2,-1,0) + z(3,1,-1) = (4,2,1)$

forma bektoriala

$\bar{u}_1 = (1,1,1)$, $\bar{u}_2 = (0,-1,2)$

$\bar{x} = (-2,5,-4) \in \mathbb{R}^3$, $\bar{u}_1$ a $\bar{u}_2$ konb. lineal. bezala

$\lambda_1\bar{u}_1 + \lambda_2\bar{u}_2 = \bar{x}$

$\lambda_1(1,1,1) + \lambda_2(0,-1,2) = (-2,5,-4)$

$$A^* = \begin{pmatrix} 1 & 0 & -2 \\ 1 & 1 & 5 \\ 1 & 2 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 2 & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

$h(A^*) = 2$

$h(A) = 2$

$$\sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 1 & -3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \end{pmatrix}$$

$\downarrow$

$\lambda_1 = 2$

$\lambda_2 = -3$

$\Downarrow$

$2(1,1,1) + -3(0,-1,2)$

$(2,2,2) + (0,3,-6)$

$(-2,5,-4)$

M2 - 3 - 015

$x \cdot \bar{a_1} + y\, \bar{a_2} + z\, \bar{a_3} = \bar{b}$

$A_z = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 2 & 0 & 3 & -4 \\ 3 & -1 & 4 & 5 \end{pmatrix}$ $\quad A \cdot \bar{x} = \bar{b} \quad$ $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 0 & 3 \\ 3 & 1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$

$x(1,2,3) + y(1,0,-1) + z(2,3,4) = (3,-4,5)$

$B_S = \{\bar{a_1}, \bar{a_2}, \bar{a_3}, \dots\}$

$\downarrow$

$h(B_S) = $ | zuke soluzioa / betez $h(B_S) = m$

$\downarrow$

oin. bekt. kop.

ELS BATERAEZINA SOLUZIO HURBILDUA

$x_1 + x_3 = 1$

$x_2 + 2x_3 = 1$

$x_1 + 2x_2 + 2x_3 = 1$

$x_1 + x_2 + 3x_3 = 1$

$A_z = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 3 & 1 \end{pmatrix}$
$\quad a_1 \; a_2 \; a_3$

$h(A) \neq h(A_z)$

$B_S^3 = \{(1,0,1,1), (0,1,2,1), (1,2,2,3)\}$

$B_S^3 = x_1(1,0,1,1) + x_2(0,1,2,1) + x_3(1,2,2,3)$

$x_1 \bar{a_1} + x_2 \bar{a_2} + x_3 \bar{a_3} = \bar{b}$

1) $B_S \to$ oinarria

2) $B_S \to B_{og}$

3) $\bar{b}$-ren $H_{L(S)} = \bar{b'}$

4) $A\bar{x} = \bar{b'}$ $\longrightarrow$ S. bateragarria

5) Sistema ebatzi $\bar{x}$: sol. hurbildua

$h(B_S^3) = 3$ oinarria da

GRAM - SCHMIDT $\Rightarrow B_{og} = \{\bar{v_1}, \bar{v_2}, \bar{v_3}\}$

$\bar{v_1} = \bar{u_1} = (1,0,1,1)$

$\bar{v_2} = \bar{u_2} - P_{v_1}\bar{u_2} = \bar{u_2} - \frac{\langle \bar{u_2}, \bar{v_1} \rangle}{\|v_1\|^2} \bar{v_1} = \bar{u_2} - \frac{3}{3} v_1 = \bar{u_2} - \bar{v_1}$

$\bar{v_3} = \bar{u_3} - P_{v_1}\bar{u_3} - P_{v_2}\bar{u_3} = \bar{u_3} - \frac{6}{3}\bar{v_1} - \frac{3}{3}\bar{v_2} = \bar{u_3} - 2\bar{v_1} - \bar{v_2}$

$\langle \bar{u_2}, \bar{v_1} \rangle = 3$

$\langle \bar{u_3}, \bar{v_1} \rangle = 6$

$\langle \bar{u_3}, \bar{v_2} \rangle = 3$

$\|\bar{v_1}\|^2 = 3$

$\|\bar{v_2}\| = 3$

$\bar{v_2} = (0,1,2,1) - (1,0,1,1) = (-1,1,1,0)$

$\bar{v_3} = (1,2,2,3) - (2,0,2,2) - (-1,1,1,0) = (0,1,-1,1)$

$B_{og} = \{(1,0,1,1), (-1,1,1,0), (0,1,-1,1)\}$

FM2 - F - 016

$A\bar{x} = \bar{b}$ $\quad \bar{b} \notin$ S = imagen $\quad \overset{=}{\bar{b}'}$

$$\bar{b}' = \sum_{i=1} P_{\bar{v}_i}\bar{b} = P_{\bar{v}_1}\bar{b} + P_{\bar{v}_2}\bar{b} + P_{\bar{v}_3}\bar{b}$$

$\bar{b} = (1,1,1,1)$

$$\frac{\langle\bar{b},\bar{v}_1\rangle}{\|\bar{v}_1\|^2}\bar{v}_1 + \frac{\langle\bar{b},\bar{v}_2\rangle}{\|\bar{v}_2\|^2}\bar{v}_2 + \frac{\langle\bar{b},\bar{v}_3\rangle}{\|\bar{v}_3\|^2}\bar{v}_3$$

$$\frac{3}{3}\bar{v}_1 + \frac{1}{3}\bar{v}_2 + \frac{1}{3}\bar{v}_2$$

$$\bar{v}_1 + \frac{1}{3}(\bar{v}_2 + \bar{v}_3)$$

$$(1,0,1,1) + \frac{1}{3}(-1,2,0,1)$$

$$(1,0,1,1) + (-1/3, 2/3, 0, 1/3)$$

$$(2/3, 2/3, 1, 4/3)$$

$\langle\bar{b},\bar{v}_1\rangle = 3$

$\langle\bar{b},\bar{v}_2\rangle = 1$

$\langle\bar{b},\bar{v}_3\rangle = 1$

$\|v_1\|^2 = 3$

$\|v_2\|^2 = 3$

$\|v_3\|^2 = 3$

$A\cdot\bar{x} = \bar{b} \Rightarrow A\bar{x} = \bar{b}'$

$\Downarrow$

$$A_Z^{*} = \left(\begin{array}{cccc|c} 1 & 0 & 1 & \cdot & 2/3 \\ 0 & 1 & 2 & \cdot & 2/3 \\ 1 & 2 & 2 & \cdot & 1 \\ 1 & 1 & 3 & \cdot & 4/3 \end{array}\right)$$

$h(A) = h(A_Z) = 3$

$$\begin{pmatrix} 1 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/3 \end{pmatrix} \quad x_1 = 1/3,\ x_2 = 0,\ x_3 = 1/3$$

$h(A) \leq 3$

$h(A_Z) \leq 4$ } SC $\to$ Det $h=3$; $\to$ Indet [redacted] $h=2$

$\downarrow$

$h(A) = h(A_Z)$

$$e = d(\bar{b},\bar{b}') = \|\bar{b} - \bar{b}'\|$$
$$= \|(1/3, 1/3, 0, -1/3)\|$$
$$= +\sqrt{(1/3)^2 + (1/3)^2 + (-1/3)^2}$$
$$= +\sqrt{\frac{1+1+1}{9}} = +\sqrt{\frac{3}{9}} = +\sqrt{1/3}$$

---

det A $\rightsquigarrow$ A es polinomio característico
$\overset{\shortparallel}{|A|}$

$$|A - \lambda I| = 0 \quad / \quad A \in \mathcal{M}_{n\times n}(\mathbb{R})\ ;\ \sigma(A) = \{\lambda_1(k_1), \lambda_2(k_2), \dots\}$$

$k_1 + k_2 + \dots = n$

$$(-1)^n \cdot P_n(\lambda) = (-1)^n \cdot (\lambda - \lambda_1)^{k_1} \cdot (\lambda - \lambda_2)^{k_2} \dots$$

$$|A - \lambda I|_{\lambda=0} \Rightarrow |A| = [(-1)^n \cdot P_n(\lambda)]_{\lambda=0}$$

$A_{3\times3}$

$\sigma(A) = \{1(k=1), -2(k=1), 3(k=1)\}$ $\quad / \quad A \in \mathcal{M}_{3\times3}(\mathbb{R})$

$P(\lambda) = [(\lambda-1)\cdot(\lambda+2)\cdot(\lambda-3)]_{\lambda=0}$ $\quad |A| = (-1)(-1)(2)(-3) = -6$

[redacted]

$\sigma(B) = \{2(k=2), -3(k=1)\}$ $\longrightarrow \{2, 2, -3\}$

$(-1)^3 P(\lambda) = (-1)^3 [(\lambda-2)^2(\lambda+3)]_{\lambda=0}$

$|A| = 12$ $\quad (-1)\ (-2)^2\ 3$

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Eigenvalues [ ]
Eigenvectors [ ]
Eigensystem [ ]

$D_{4\times4}$ / $\sigma(D) = \{1, -3, 2, 0\}$

$(-1)^4 P(\lambda) = [1\ (\lambda-1)(\lambda+3)(\lambda-2)(\lambda-0)]_{\lambda=0}$

$|A| = 1\cdot(-1)\cdot 3\cdot(-2)\cdot 0 = 0$

$S = \left\{ \begin{pmatrix} a+b+3c & 2a-b \\ -a-c & a+2b+5c \end{pmatrix} / a,b,c \in \mathbb{R} \right\}$

$a(1,2,-1,1) + b(1,-1,0,2) + c(3,0,-1,5)$

$h(G_S) = h \begin{pmatrix} 1 & 2 & -1 & 1 \\ 1 & -1 & 0 & 2 \\ 3 & 0 & -1 & 5 \end{pmatrix} = h \begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & -3 & 1 & 1 \\ 0 & -6 & 2 & 2 \end{pmatrix} = h \begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & -3 & 1 & 1 \\ \cancel{0} & \cancel{-3} & \cancel{1} & \cancel{1} \end{pmatrix} - h$

Bonus: but eten dimentsion kalkulate

$G_S = \left\{ \overset{\bar{e_1}}{\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}}, \overset{\bar{e_2}}{\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}}, \overset{\bar{e_3}}{\begin{pmatrix} 3 & 0 \\ 1 & 5 \end{pmatrix}} \right\}$ $\quad B_S = \{\bar{e_1}, \bar{e_2}\}$

$a\bar{e_1} + b\bar{e_2} + c\bar{e_3}$

$\bar{u} = (1, 2, -1)$
$\bar{v} = (0, 1, 3)$

$<\bar{u},\bar{v}> = 0 + 2 - 3 = -1$

$||\bar{u}|| = +\sqrt{<\bar{u},\bar{u}>} = \sqrt{1^2+2^2+(-1)^2} = \sqrt{6}$

$||\bar{v}|| = \sqrt{0+1+9} = \sqrt{10}$

$d(\bar{u},\bar{v}) = ||\bar{u}-\bar{v}|| = ||(1,1,-4)||$

$P_{\bar{v}}\bar{u} = \frac{<\bar{u},\bar{v}>}{||\bar{v}||^2}\cdot\bar{v}$

$P_{\bar{u}}\bar{v} =$

$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 & 0 \\ 0 & -1-\lambda & 0 & -1 \\ 0 & 0 & -1-\lambda & -1 \\ 0 & -1 & -1 & -2-\lambda \end{vmatrix} = (1-\lambda) \begin{vmatrix} -1-\lambda & 0 & -1 \\ 0 & -1-\lambda & -1 \\ -1 & -1 & -2-\lambda \end{vmatrix} = (-1)^3(1-\lambda) \begin{vmatrix} 1+\lambda & 0 & 1 \\ 0 & 1+\lambda & 1 \\ 1 & 1 & 2+\lambda \end{vmatrix}$

<< LinearAlgebra`Orthogonalization`
pr. Characteristic Polynomial

$[(-1-\lambda)(-1-\lambda)(-2-\lambda)] - [(-1-\lambda)+(-1-\lambda)]$

$(1+\overset{2\lambda}{\cancel{\lambda+\lambda}}+\lambda^2)(-2-\lambda) - (-2-2\lambda)$

$-2-4\lambda-2\lambda^2-\lambda-2\lambda^2-\lambda^3+2+2\lambda$

$-\lambda^3-4\lambda^2-3\lambda$

$\Downarrow$

$(1-\lambda)(-\lambda^3-4\lambda^2-3\lambda)$

$(1+\lambda)(1+\lambda)(2+\lambda) - (1+\lambda) - (1+\lambda)$

$(1+\lambda+\lambda+\lambda^2)(2+\lambda) - 2(1+\lambda)$

$\cancel{2}+4\lambda+2\lambda^2+\lambda+2\lambda^2+\lambda^3-\cancel{2}-2\lambda$

$\lambda^3+4\lambda^2+3\lambda$

$\Downarrow$

$(\lambda-1)(\lambda^3+4\lambda^2+3\lambda)$

$(\lambda-1)\left[(1+\lambda)^2(2+\lambda) - (1+\lambda) - (1+\lambda)\right] \Rightarrow (\lambda-1)(1+\lambda)\left[(1+\lambda)(2+\lambda) - 2\right]$

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$$(\lambda-1)(1+\lambda)\left[\lambda^2+3\lambda+2-2\right] \Rightarrow (\lambda-1)(\lambda+1)\,\lambda\,(\lambda+3)=0$$

$$\boxed{\begin{array}{l}\lambda=1\\ \lambda=-1\\ \lambda=0\\ \lambda=-3\end{array}}$$

ELS - HOMOGENEOA

↓

Forma matrizialea

$$(A-\lambda I)X_{n\times 1}=(0)_{n\times 1}$$

$$X=\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}$$

1) GAUSS

2) ROUCHÉ-FROB $\longrightarrow \begin{cases}\text{Er. neg}\\ \text{Er. eskt} \rightarrow \dim S(\lambda)\end{cases}$

3) ORDEZKATU BAT KALKULATU

$$(A_+)_{\lambda=0}=\left(\begin{array}{cccc|c}1&0&0&0&0\\0&1&0&1&0\\0&0&1&1&0\\0&1&1&2&0\end{array}\right)$$

$$\sim\left(\begin{array}{cccc|c}1&0&0&0&0\\0&1&0&1&0\\0&0&1&1&0\end{array}\right)$$

$$h(A)=h(A_+)=3<4\begin{cases}\text{Ez.ngz}\rightarrow 3\\ \text{Ez.askt}\rightarrow 1\end{cases}$$

$$\begin{aligned}x_1&=0\\x_2+x_4&=0\\x_3+x_4&=0\end{aligned}\qquad\begin{aligned}x_1&=0\\x_2&=x_3=-x_4\end{aligned}$$

dimentsioa $\Rightarrow 1$